$$\int \sin^4(x)\cos^2(x)\,dx$$

Compute $\int \sin^4(x)\cos^2(x)\,dx$.

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$$\int \sin^4 x \cos^2 x \, dx$$

$$\sin^4(x) = \left(\sin^2(x)\right)^2 = \left(\frac{1-\cos 2x}{2}\right)^2$$

$$\cos^2(x) = \frac{1+\cos 2x}{2}$$

$$= \int (\sin^2 x \cos^2 x)\sin^2 x \, dx$$

$$= \int \left(\frac{\sin 2x}{2}\right)^2 \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \frac{1}{8}\int \left(\frac{1-\cos 4x}{2}\right)(1-\cos 2x)\, dx$$

$$= \frac{1}{16}\int 1 - \cos 4x - \cos 2x + \cos 2x \cos 4x \, dx$$

$$= \frac{1}{16}\int 1 - \cos 4x - \cos 2x + (-\cos 2x + 2\cos^3 2x)\, dx$$

$$= \frac{1}{16}\left(x - \frac{\sin 4x}{4} - \frac{2\sin 2x}{2}\right) + \frac{1}{16}\left(\sin 2x - \frac{\sin^3 2x}{3}\right) + C$$

$$= \frac{1}{16}\left(x - \frac{\sin 4x}{4} - \frac{\sin^3 2x}{3} + C\right)$$

$$\cos 4x = 2\cos^2 2x - 1$$

$$\Rightarrow \cos 2x \cos 4x = 2\cos^3 2x - \cos 2x$$

$$\int 2\cos^3 2x \, dx$$

$$= 2\int (1-\sin^2 2x)\cos 2x \, dx$$

$$= 2\int (1-u^2)\cdot \frac{1}{2}\, du$$

$$= u - \frac{u^3}{3} + C$$

$$= \sin 2x - \frac{\sin^3 2x}{3} + C$$